Three or Four Eggs in a Quiche?

A vignette on creating math problems.

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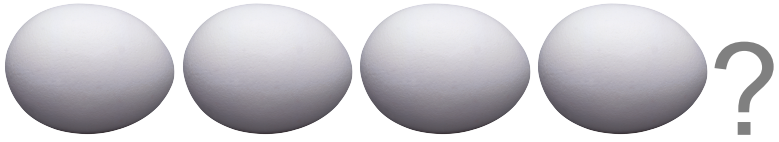

In the article “Bureaucrat's Math Makes Dizzy Dozen,” written by P. Rolly and J. Jacobsen- Wells and published in The Salt Lake Tribune on 11 October 2002 (Article ID: 100DF2EEC6A2847B) we read:

> The menu at the Coffee Garden at 900 East and 900 South in Salt Lake City has included a scrumptious selection of quiche for about 10 years. The recipe calls for four fresh eggs for each quiche. A Salt Lake County Health Department inspector paid a visit recently and pointed out that research by the Food and Drug Administration (FDA) indicates that one in four eggs carries salmonella bacterium, so restaurants should never use more than three eggs when preparing quiche.

On your own, or with a partner, discuss and answer the following questions:

Review and warm-up (this part can be shortened or expanded, as needed):

(1) In your own words, explain what the probability (of some event occurring) is and how it is expressed numerically.

(2) What does it mean to say that the probability of an event is zero? One? 50%?

(3) You toss a coin once. What is the probability that you obtain a head? What assumption(s) did you make in answering this question?

(4) You toss a coin twice. What is the probability of getting two heads? What assumption(s) did you make in answering this question?

(5) You toss a coin three times. In how many different ways can you obtain exactly two heads? What is the probability of obtaining exactly two heads?

Now re-read the newspaper article.

(6) Try to articulate, without performing any calculations, the fallacy of the inspector’s argument.

Discus and answer the following questions:

(7) In a box of dozen eggs, how many are likely to carry salmonella bacterium? In five boxes of dozen eggs each, how many are likely to carry salmonella bacterium?

(8) What is the probability that a randomly selected egg from a box of dozen eggs carries salmonella bacterium?

(9) What is the probability that a randomly selected egg from a box of dozen eggs does NOT carry salmonella bacterium?

We introduce a useful bit of notation. By X we denote the number of eggs (from those that we select) that carry salmonella bacterium; as usual, by p we denote the probability. The answer to (8) can be denoted by $p(X=1)$. Likewise, we use $p(X=0)$ for our answer to question (9).

(10) Using simple tools (pencil, paper, etc.) design an experiment that would simulate the action of picking three eggs in a row from a box of dozen eggs. Run your experiment 20 times, record the outcomes and use it to figure out the probability of picking three eggs in a row which do not carry salmonella.

(11) For the case of picking three eggs in a row from a box of dozen eggs, compute the probability $p(X=0)$; i.e., find the probability that none of the three picked eggs carry salmonella. Make sure to identify the assumptions you used in your calculations, and comment on whether or not they are reasonable in the context of the quiche story.

(12) Compare your answer to (10) with your simulation result. Did you expect the two to be equal? Why or why not?

(13) Explain how you could make the result of your simulation match better your answer in (11). If you know programming, write a code that would simulate picking three eggs in a row from a box of dozen eggs, and run the code (how many times?). Does the outcome of your computer simulation support your answer?

(14) Compute $p(X=0)$ in the case of picking four eggs in a row from a box of dozen eggs. Adjust your computer simulation program and test your answer.

Equipped with your knowledge of probability, go back to the article.

(15) If you were the inspector, based on your calculations and the FDA statsitics, what answer would you give to the owner of the restaurant?

Next, we consider a few extensions.

Assume that we are picking three eggs in a row from a box of dozen eggs.

(16) In how many different ways can we pick one egg with salmonella? What is the probability $p(X=1)$?

(17) In words, state which probability is represented by $p(X=3)$. Compute $p(X=2)$, $p(X=3)$, and $p(X=4)$.

(18) Represent the probabilities you obtained in (11), (16) and (17) in a suitably chosen diagram. Think about which quantity to put on the *x*-axis and which one would be appropriate for the *y*-axis.

(19) Add up the probabilities you obtained in (11), (16) and (17). Does it make sense? Why, or why not?

Now assume that we are picking four eggs in a row from a box of dozen eggs.

(20) Find the probabilities $p(X=0)$, $p(X=1)$, $p(X=2)$, $p(X=3)$, and $p(X=4)$, represent them in a diagram as in (18)) and add them up. Does the sum make sense?

Further extensions.

(21) Assume that there are three boxes of dozen eggs each, labeled A, B and C. You know that in box A one in four eggs has salmonella, in box B one in three eggs has salmonella, and in box C one in two eggs has salmonella. You pick one egg from each box. What is $p(X=0)$? (Remember that X counts the number of eggs with salmonella). Find $p(X=1)$, $p(X=2)$, $p(X=3)$? State all assumptions which you made in your calculations.

(22) Design an experiment (including suitable defining the quantity X) for which the probabilities $p(X=0)$, $p(X=1)$, $p(X=2)$, $p(X=3)$, and $p(X=4)$, are the same as in (20).

Now the second part of the article: the store manager asks the inspector whether throwing three randomly chosen eggs from each dozen, and then making 4-eggs quiches from the remaining eggs would work (i.e., would guarantee that the quiche is made of non-salmonella eggs only).

(23) Design an experiment that would simulate this situation (taking a box of dozen eggs, throwing away three randomly selected eggs and then making a quiche from four randomly selected eggs among the nine left in the box). Repeat your experiment 15 times and figure out the distribution of X; i.e., find the probabilities $p(X=0)$, $p(X=1)$, $p(X=2)$, $p(X=3)$, and $p(X=4)$.

(24) Compare your answers with other groups' answers.

(25) Based on your calculations, how would you answer the restaurant owner's question?

Challenge (could be a project topic on its own):

(26) Compute the exact probabilities $p(X=0)$, $p(X=1)$, $p(X=2)$, $p(X=3)$, and $p(X=4)$ defined in (23).

NOTE: Please send me an email (lovric@mcmaster.ca) and I will be happy to send you answers and solutions to all questions.